Fingers and toes

Humans have always used their fingers for counting. This explains the different increments used in various numeric systems.
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Some cultures used the five fingers of one hand or ten fingers of both hands, while others used the ten toes in addition to the fingers. The numeric system in ancient Egypt, for example, was based on increments of ten, while the Maya and Aztec counted in increments of 20. The French number quatre-vingt (four times 20) meaning 80, and a similar form used in Danish, are traces of this numeric system that uses increments of 20. The Babylonians were an exception: they used increments of 60. The graduation of hours into 60 minutes of 60 seconds each and angular units are still used.
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Numeric systems

Humans created the concept of mathematical numbers to describe quantities such as the size of an animal herd with a limited amount of effort, and to put these quantities in order.
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The development of numeric systems was preceded by a lengthy process. This is due to the level of abstraction required in associating the same terms with the same quantities of objects in order to find a
generally binding description of quantities; for example not only three cups are associated with the number three, but also three spoons, three apples, and so on. Therefore, the number “three”
is the result of an abstraction process, which took some time to establish in various cultures. Today still,
some indigenous people only use small numbers, which they associate with particular, tangible objects.
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EUCLID’S ELEMENTS

Euclid’s manuscript The Elements is one of the most significant sources of Greek mathematics. It is the first systematic summary of fundamental geometry and arithmetics. Euclid derives the characteristics of geometric objects and natural numbers from axioms (basic statements that do not require proof)
and covers geometric algebra, proportions, primes, divisibility of numbers, and the method of exhaustion. *Euclid of Alexandria, Greek mathematician, around 365 to 300 B.C.*

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High point of mathematics

Mathematical historians concentrate mainly on ancient Greece, as the Romans were predominantly preoccupied with architecture and law. The Greek philosopher Thales of Miletus is considered the first mathematician who demonstrated by logical argumentation that his theorems were valid. He was born
around 620 B.C. The Greeks have passed down formulas, laws, and rules about geometry which are still valid today. Not only did they prove their own formulas but also the ones previously

stated by the
Babylonians and Egyptians.
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In contrast to the Babylonian, Egyptian, and Greek civilizations, the history of Chinese mathematics dates back to 4,000 years ago. The Jiuzhang Suanshu or The Nine Chapters on the Mathematical Art is the most famous Chinese textbook on mathematics of all time. It originates from about A.D. 100 and contains solutions to problems as well as algorithms. Additional texts about the proof of algorithms were written up to the 13th century A.D., the high point of mathematics in China.
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The first Idea of mathematics

Civilizations in the ancient Middle East, China, India, and ancient America explored mathematical problems long before

mathematics became a discipline in itself. (□□□□□□□ □□□□ □□□□□, □□□, □□□□ □□ □□□□□□□ □□□□□□□ □□ □□□□□□□□ □□ □□□□ □□ □□□□ □□ □□□ □□ □□□□□□□ □□□□ □□ □□□□ □□□□ □□□□□□ □□□□□□□□ □□ □□□ □□ □□□)

The first approaches to mathematics as we know it today date back about 5,000 years (before 3000 B.C.), from Mesopotamia. They generated the Sumerian, Babylonian, and Assyrian cultures. The most important mathematical scripts originate from the old Babylonian period (around 2000 B.C.). (□□□□ □□ □□□□ □□□ □□□□ □□□□□□□□□ □□□□ □□ □□ □□ □□□□□ □□□, □□□□ 5,000 □□□ □□□□ (3000 □□□ □□□□□ □□ □□□□) □□□□□□□□□□□□ □□ □□□ □□□ □□□□□□□□ □□□□□□□□, □□□□□□□□□□ □□ □□□□□□□ □□□□□□□□□□□ □□ □□□□ □□□□□ □□□□ □□□□□□□□□□ □□□□□□ □□□□□□□ □□□□□□ □□□□□□□□□□ □□□ (□□□□ 2000 □□□ □□□□□□) □□ □□□□□□□ □□□ □□□□)

These included formulas for calculating areas, volume, approximate values for the number pi, calculation of V2 with an accuracy of six decimal digits and references to the Pythagorean theorem about a right-angled triangle. (□□□□□ □□□□□□□□□□, □□□□, □□□□□□ □□□ □□ □□□□□□□□ □□□, □□ □□□□□ □□□□□ □□ □□□□□□ □□ □□□ V2 □□ □□□□ □□ □□□□□ □□□□□□□ □□ □□□□ □□□ □□□□□□□□□ □□□□□□ □□ □□□□□□ □□ □□□ □□□□□ □□□□□ □□□)

The most important Egyptian sources that remain from the Middle Kingdom (around 2100 to around 1790 B.C.) are two papyrus manuscripts and one leather script proving that basic calculation methods, fractions, geometry, and the number pi were already known in ancient Egypt. Despite these advances, neither of the two cultures has passed down any mathematical proofs.
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